III Congresso Brasileiro de Jovens Pesquisadores
em Matemática Pura, Aplicada e Estatística

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Session: Teoria de Grupos e Aplicações

Organized by Alex Cardoso Dantas (Universidade Tecnológica Federal do Paraná)
Emerson Ferreira de Melo (Universidade de Brasília)
and Raimundo de Araújo Bastos Júnior (Universidade de Brasília)

Schedule

Wednesday, December 12

9:00 - 9:30	Opening
9:30 - 10:30	Plenary talk 1
10:30 - 11:00	Coffee break
11:00 - 11:30	Igor dos Santos Lima (UnB) *On the number of cyclic subgroups of a finite group*
11:30 - 12:00	Jhone Caldeira Silva (UFG) *Ações por automorfismos com centralizadores nilpotentes em grupos finitos*
12:00 - 13:30	Lunch
13:30 - 14:30	Plenary talk 2
14:40 - 15:10	DANILO SANÇÃO DA SILVEIRA (UFG) *Pontos fixos de automorfismos de grupos e condições de Engel*
15:10 - 15:40	Agenor Freitas de Andrade (IFG) *Bounding the order of the nilpotent residual*
15:40 - 16:10	Alex Carrazedo Dantas (UnB) *Representações auto-similares de produtos entrelaçados de grupos abelianos*
16:10 - 16:40	John William MacQuarrie (UFMG) *Recognizing permutation modules*
16:40 - 17:10	Coffee break
17:10 - 18:40	Round Table

Thursday, December 13

9:00 - 10:00	Plenary talk 3
10:00 - 10:30	Coffee break
10:30 - 11:00	Raimundo de Araújo Bastos Júnior (UnB) *Condições de Engel em grupos*
11:00 - 11:30	Francismar Ferreira Lima (UTFPR) *Finiteness Conditions and some Conjectures*
11:30 - 12:00	Tiago Luiz Ferrazza (Universidade Positivo) *Morita Equivalence of Partial Hopf Actions*
12:00 - 13:30	Lunch
13:30 - 14:30	Plenary talk 4
14:40 - 15:10	Emerson Ferreira de Melo (UnB) *Automorfismos coprimos de grupos finitos*
15:10 - 15:40	Mohsen Amiri (UFAM) *Sum of element orders in a finite group*
15:40 - 16:10	Luís Augusto de Mendonça (Unicamp) *Algebraic properties of groups of pure symmetric automorphisms of RAAGs*
16:40 - 17:10	Coffee break
20:00 - 0:00	Social dinner

Friday, December 14

9:00 - 10:00	Plenary talk 5
10:00 - 10:30	Coffee break
13:30 - 14:30	Plenary talk 6
16:40 - 17:10	Coffee break
17:10 - 18:40	Assembly

Abstracts

1. *Speaker:* **Igor dos Santos Lima**
   Affiliation: Universidade de Brasília
   Title: ***On the number of cyclic subgroups of a finite group***

   This is a joint work with Martino Garonzi (UnB) accepted for publication in the Bulletin of the Brazilian Mathematical Society, New Series (2018). Let G be a finite group and let $c(G)$ be the number of cyclic subgroups of G. We study the function $\alpha(G) = c(G)/|G|$. We explore its basic properties and we point out a connection with the probability of commutation. For many families F of groups we characterize the groups $G \in F$ for which $\alpha(G)$ is maximal and we classify the groups G for which $\alpha(G) > 3/4$. We also study the number of cyclic subgroups of a direct power of a given group deducing an asymptotic result and we characterize the equality $\alpha(G) = \alpha(G/N)$ when G/N is a symmetric group.

2. *Speaker:* **Jhone Caldeira Silva**
   Affiliation: Universidade Federal de Goiás
   Title: ***Ações por automorfismos com centralizadores nilpotentes em grupos finitos***

   Apresentaremos resultados a respeito de ações de grupos. Abordaremos automorfismos com centralizadores nilpotentes agindo sobre grupos finitos. Consideramos um grupo A agindo por automorfismos sobre outro grupo G e denotamos por $C_G(A)$ o centralizador de A em G, que é o subgrupo dos pontos fixos da ação de A sobre G. É conhecido que a estrutura de $C_G(A)$ tem forte influência sobre a estrutura de G. Em particular, muito têm sido investigadas situações em que um grupo de Frobenius age por automorfismos sobre um grupo finito G. Aqui, apresentamos alguns exemplos deste fenômeno, contando um pouco da história do desenvolvimento dos resultados referentes ao problema, até a apresentação de novos resultados obtidos. Mais precisamente traremos maiores detalhes a respeito do seguinte: sejam A um grupo finito e M um subgrupo normal de A. Suponha que todos os elementos de $A \setminus M$ têm ordem prima p. Se A age sobre um p'-grupo G de tal modo que $C_G(M) = 1$ e $C_G(x)$ é nilpotente para todo $x \in A \setminus M$, então G é também nilpotente. Ainda, no caso especial em que A é um p-grupo e $C_G(x)$ tem classe de nilpotência limitada para todo $x \in A \setminus M$, aplicamos métodos lineares para mostrar que G tem classe de nilpotência limitada. A metodologia para alcançar os principais resultados passa pelos grupos de Frobenius, pelos automorfismos splitting e por técnicas sobre anéis e álgebras de Lie.

3. *Speaker:* **DANILO SANÇÃO DA SILVEIRA**
   Affiliation: Universidade Federal de Goiás
   Title: ***Pontos fixos de automorfismos de grupos e condições de Engel***

   Sejam q um número primo e A um adequado q-grupo abeliano elementar agindo coprimamente sobre um grupo finito ou profinito G. Mostramos que se para cada $a \in A^{\#}$ elementos nos centralizadores $C_G(a)$ satisfazem alguma condição de Engel, então o grupo todo G satisfaz uma condição de Engel similar.

4. *Speaker:* **Agenor Freitas De Andrade**
   Affiliation: IFG
   Title: ***Bounding The Order Of The Nilpotent Residual***

The coprime commutators γ_k^* and δ_k^* in a finite group G are defined as follows. Every element of G is both a γ_1^*-commutator and a δ_0^*-commutator. Assume that $k \geq 2$ and let S be the set of all elements of G that are powers of γ_{k-1}^*-commutators. An element g is a γ_k^*-commutator if there exist $a \in S$ and $b \in G$ such that $g = [a, b]$ and $(|a|, |b|) = 1$. For $k \geq 1$ let T be the set of all elements of G that are powers of δ_{k-1}^*-commutators. The element g is a δ_k^*-commutator if there exist $a, b \in T$ such that $g = [a, b]$ and $(|a|, |b|) = 1$. For every $k \geq 1$ the subgroup generated by γ_k^*-commutators is precisely $\gamma_\infty(G)$ and for every $k \geq 0$ the subgroup generated by δ_k^*-commutators is precisely the kth term of the lower Fitting series of G. The main result of this article is the following theorem. Let m be a positive integer and G a finite group. Let $X \subset G$ be either the set of all γ_k^*-commutators for some fixed $k \geq 2$ or the set of all δ_k^*-commutators for some fixed $k \geq 1$. Suppose that the size of a^X is at most m for any $a \in G$. Then the order of $\langle X \rangle$ is (k, m)-bounded.The coprime commutators γ_k^* and δ_k^* in a finite group G are defined as follows. Every element of G is both a γ_1^*-commutator and a δ_0^*-commutator. Assume that $k \geq 2$ and let S be the set of all elements of G that are powers of γ_{k-1}^*-commutators. An element g is a γ_k^*-commutator if there exist $a \in S$ and $b \in G$ such that $g = [a, b]$ and $(|a|, |b|) = 1$. For $k \geq 1$ let T be the set of all elements of G that are powers of δ_{k-1}^*-commutators. The element g is a δ_k^*-commutator if there exist $a, b \in T$ such that $g = [a, b]$ and $(|a|, |b|) = 1$. For every $k \geq 1$ the subgroup generated by γ_k^*-commutators is precisely $\gamma_\infty(G)$ and for every $k \geq 0$ the subgroup generated by δ_k^*-commutators is precisely the kth term of the lower Fitting series of G. The main result of this article is the following theorem. Let m be a positive integer and G a finite group. Let $X \subset G$ be either the set of all γ_k^*-commutators for some fixed $k \geq 2$ or the set of all δ_k^*-commutators for some fixed $k \geq 1$. Suppose that the size of a^X is at most m for any $a \in G$. Then the order of $\langle X \rangle$ is (k, m)-bounded.The coprime commutators γ_k^* and δ_k^* in a finite group G are defined as follows. Every element of G is both a γ_1^*-commutator and a δ_0^*-commutator. Assume that $k \geq 2$ and let S be the set of all elements of G that are powers of γ_{k-1}^*-commutators. An element g is a γ_k^*-commutator if there exist $a \in S$ and $b \in G$ such that $g = [a, b]$ and $(|a|, |b|) = 1$. For $k \geq 1$ let T be the set of all elements of G that are powers of δ_{k-1}^*-commutators. The element g is a δ_k^*-commutator if there exist $a, b \in T$ such that $g = [a, b]$ and $(|a|, |b|) = 1$. For every $k \geq 1$ the subgroup generated by γ_k^*-commutators is precisely $\gamma_\infty(G)$ and for every $k \geq 0$ the subgroup generated by δ_k^*-commutators is precisely the kth term of the lower Fitting series of G. The main result of this article is the following theorem. Let m be a positive integer and G a finite group. Let $X \subset G$ be either the set of all γ_k^*-commutators for some fixed $k \geq 2$ or the set of all δ_k^*-commutators for some fixed $k \geq 1$. Suppose that the size of a^X is at most m for any $a \in G$. Then the order of $\langle X \rangle$ is (k, m)-bounded.The coprime commutators γ_k^* and δ_k^* in a finite group G are defined as follows. Every element of G is both a γ_1^*-commutator and a δ_0^*-commutator. Assume that $k \geq 2$ and let S be the set of all elements of G that are powers of γ_{k-1}^*-commutators. An element g is a γ_k^*-commutator if there exist $a \in S$ and $b \in G$ such that $g = [a, b]$ and $(|a|, |b|) = 1$. For $k \geq 1$ let T be the set of all elements of G that are powers of δ_{k-1}^*-commutators. The element g is a δ_k^*-commutator if there exist $a, b \in T$ such that $g = [a, b]$ and $(|a|, |b|) = 1$. For every $k \geq 1$ the subgroup generated by γ_k^*-commutators is precisely $\gamma_\infty(G)$ and for every $k \geq 0$ the subgroup generated by δ_k^*-commutators is precisely the kth term of the lower Fitting series of G. The main result of this article is the following theorem. Let m be a positive integer and G a finite group. Let $X \subset G$ be either the set of all γ_k^*-commutators for some fixed $k \geq 2$ or the set of all δ_k^*-commutators for some fixed $k \geq 1$. Suppose that the size of a^X is at most m for any $a \in G$. Then the order of $\langle X \rangle$ is (k, m)-bounded.

5. *Speaker:* **Alex Carrazedo Dantas**
   Affiliation: Universidade de Brasília - UnB
   Title: ***Representações auto-similares de produtos entrelaçados de grupos abelianos***

   Nessa apresentação, veremos que se existir uma representação auto-similar de um produto entrelaçado $G = B \wr X$ de grupos abelianos B e X, onde X é livre de torção, então B é de torção com expoente limitado. Em particular, o Lamplighter grupo $\mathbb{Z} \wr \mathbb{Z}$ não é auto-similar. Trabalho em conjunto com Said Najati Sidki.

6. *Speaker:* **John William MacQuarrie**
   Affiliation: UFMG
   Title: ***Recognizing permutation modules***

   Let G be a finite p-group and $\mathbb{Z}_p$ the ring of p-adic integers. We consider permutation $\mathbb{Z}_pG$-modules – that is, modules that are free over $\mathbb{Z}_p$ and having a $\mathbb{Z}_p$-basis preserved by the action of G. A surprisingly difficult question is: given a module, how do I know if it is a permutation module? In 1988, A. Weiss proved a theorem that recognizes a module as being a permutation module if it satisfies certain conditions as a module for smaller subgroups. I will show a certain generalization of this theorem, wherein we recognize our module as a permutation module when weaker conditions for the smaller subgroups are satisfied.

7. *Speaker:* **Raimundo de Araújo Bastos Júnior**
   Affiliation: Universidade de Brasília
   Title: ***Condições de Engel em grupos***

   Apresentar critérios de nilpotência (local) para grupos infinitos a partir de condições tipo-Engel.

8. *Speaker:* **Francismar Ferreira Lima**
   Affiliation: Universidade Tecnológica Federal do Paraná (UTFPR)
   Title: ***Finiteness Conditions and some Conjectures***

   There is a conjecture called n-$(n+1)$-$(n+2)$ Conjecture, that we call Homotopic n-$(n+1)$-$(n+2)$

Conjecture in this work. This claims that, for $n \geq 0$, given two short exact sequences of groups $N_1 \hookrightarrow G_1 \overset{\pi_1}{\twoheadrightarrow} Q$ and $N_2 \hookrightarrow G_2 \overset{\pi_2}{\twoheadrightarrow} Q$, if N_1 is of homotopic type F_n, both G_1 and G_2 are of homotopic type F_{n+1} and Q is of homotopic type F_{n+2}, then the fiber product P of π_1 and π_2 is of homotopic type F_{n+1}. Related to the latter conjecture there is another one called Virtual Surjection Conjecture, that we also call Homotopic in this work. This claims that, for $n \geq 2$, given $G_1, \ldots, G_k$ groups of homotopical type F_n, where $n \leq k$, and $P \subseteq G_1 \times \ldots \times G_k$ a subgroup that virtually surjects on every n factors, i.e. for every $1 \leq i_1 < \ldots < i_n \leq k$ the image of P under canonical projection $P \to G_{i_1} \times \ldots \times G_{i_k}$ has finite index, then P is of type F_n. These conjectures are unsolved until now, but Benno Kuckuck proved some interesting related results in 2012. Motivated by Kuckuck's work, we have proposed the **Homological** n-$(n+1)$-$(n+2)$ Conjecture and **Homological** Virtual Surjection Conjecture that the assertions are the same of the conjectures above replacing F_n with FP_n. We have proved analogous results to Kuckuck's results, but using spectral sequences in some of then. Furthermore the work here is quite different from Kuckuck's work because our groups are not finitely presented in general. In special we have proved Homological 1-2-3 Conjecture when Q is finitely presented and Homological Virtual Surjection Conjecture when $n = 2$ (Homological VSP Criterion). This is a work join with Dessislava H. Kochloukova.

9. *Speaker:* **Tiago Luiz Ferrazza**
   Affiliation: Universidade Positivo
   Title: ***Morita Equivalence of Partial Hopf Actions***

   In this work, we extend the definition and results about Morita equivalence of partial group actions on idempotent algebras to the Hopf case. First, we define the respective notion of Morita equivalence of partial Hopf actions, then we show some results: the regularity of the definition, the Morita equivalence between the respective partial smash product, the existence of a globalizable Morita equivalent partial action for any symmetrical partial action and the Morita equivalence between the global actions of the respective standard (minimal) globalizations. Also, we comment on some peculiarities when the Hopf algebra considered is the group algebra $\Bbbk G$.

10. *Speaker:* **Emerson Ferreira de Melo**
    Affiliation: Universidade de Brasília
    Title: ***Automorfismos coprimos de grupos finitos***

    Seja q um primo e A um q-grupo finito de expoente q agindo como grupo de automorfismos sobre um q'- grupo finito G. Nessa palestra apresentaremos resultados recentes que mostram que propriedades do residual nilpotente e do subgrupo de Fitting de G estão relacionadas com as respectivas propriedades do residual nilpotente e do subgrupo de Fitting dos centralizadores dos elementos de A em G.

11. *Speaker:* **Mohsen Amiri**

Affiliation: Ufam
Title: ***Sum of element orders in a finite group.***

For a finite group G, let $\Psi(G)$ denote the sum of element orders of G. In this work shop we give a short survey in the properties of this function and it's relation with the structure of the finite group G.

12. *Speaker:* **Luís Augusto de Mendonça**
Affiliation: Universidade Estadual de Campinas
Title: ***Algebraic properties of groups of pure symmetric automorphisms of RAAGs***

We consider the group of pure symmetric automorphisms of RAAGs, that is, those automorphisms that act by conjugation on each of the usual generators of the RAAG. It may happen that these groups are again RAAGs (with different defining graphs), and results by Koban and Pigott explain exactly when this is the case. We study the properties of Koszulness (for the associated Lie algebras) and polyfreeness of our groups. These properties hold in general for RAAGS, but, as we will see, they do not hold in general for groups of pure symmetric automorphisms of RAAGs. This is work in progress by the speaker and Conchita Martínez Pérez (Universidad de Zaragoza).